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Facies Constrained Elastic Full Waveform Inversion

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Summary

Current efforts to utilize full waveform inversion (FWI) as a tool beyond acoustic imaging applications, for example for reservoir analysis, face inherent limitations on resolution and also on the potential trade-off between elastic model parameters. Adding rock physics constraints does help to mitigate these issues. However, current approaches to add such constraints are based on averaged type rock physics regularization terms. Since the true earth model consists of different facies, averaging over those facies naturally leads to smoothed models. To overcome this, we propose a novel way to utilize facies based constraints in elastic FWI. A so-called confidence map is calculated and updated at each iteration of the inversion using both the inverted models and the prior information. The numerical example shows that the proposed method can reduce the cross-talks and also can improve the resolution of inverted elastic properties.

Introduction

Full waveform inversion (FWI), in principle, aims to utilize all the information in the recorded data to reconstruct the subsurface structure and to estimate the elastic and/or acoustic parameters. To better simulate wave propagation, depending on the efficiency or accuracy requirements, pseudo-acoustic, elastic and viscoelastic equations are used for the forward modeling engine in FWI. With the current availability and improvements in computational power, solving these complex equations is becoming more and more practical. However, more complex equations require more parameters to describe the real earth and inevitably introduce more null space.

Estimating elastic parameters, such as P-wave velocity, S-wave velocity, and density is an ongoing cause of the seismic exploration community. In current practice, due to an inherent crosstalk between parameters, for example, P-wave velocity and density, the density model is not usually updated at all to reduce the nonlinearity (null space) of the inversion which ultimately leads to a better convergence. However, to get a better understanding of the subsurface, multiparameter inversion is necessary. Other common ways to reduce the null space in multiparameter inversion are better parametrization (Operto et al., 2013; Oh and Alkhalifah, 2016) and incorporation of *a priori* information to constrain the inversion. Utilizing *a priori* information in the form of preconditioning or regularization has been shown to efficiently reduce the null space (Asnaashari et al., 2013).

In classic AVO inversion, however, a more advanced type of constraints based on facies has proved to be very effective to optimize the seismic inversion (Zabihi Naeini and Exley, 2017). Zabihi Naeini et al. (2016) discussed the main components of FWI as a potential reservoir characterization tool and one of their suggestions was to use facies based rock physics constraints in FWI. In this paper, we, therefore, utilize one such facies based constraint in FWI. We assume that the inverted models adhere to a Gaussian distribution (Tarantola, 2005) and, iteratively, based on the prior information, a so-called facies confidence map is calculated and used as a regularization term in inversion.

Theory

Our proposed misfit function contains a standard data misfit term, a smoothed Total Variation (TV) regularization term and a facies-based regularization term, as follows

$$J(\mathbf{m}) = J_d(\mathbf{m}) + \alpha J_{TV}(\mathbf{m}) + \beta J_{prior}(\mathbf{m}), \tag{1}$$

where α and β control the contribution from the penalty terms, and $\mathbf{m}$ denotes a vector of model parameters, which are the P-wave, S-wave velocities and density. The standard data misfit is given by

$$J_d = \parallel \mathbf{W}_d(\mathbf{d}^{pre} - \mathbf{d}^{obs}) \parallel^2, \tag{2}$$

where $\mathbf{d}$, with the corresponding superscripts, denote the vectors of multicomponent data, and $\mathbf{W}_d$ is a weighting operator applied to the data, $\mathbf{W}_d = \sigma_d \mathbf{I}$. Here, σ_d is the standard deviation of the predicted data. We use a smoothed TV as a penalty in the objective function, as follows

$$J_{TV} = \int \sqrt{\varepsilon^2 + \parallel \nabla \mathbf{m} \parallel^2} dx, \tag{3}$$

where ε mitigates the singularities in the gradient. The last term in equation 1 utilizes an *a priori* given by the facies constraint, as a penalty, as follows

$$J_{prior} = \parallel \mathbf{W}_m(\mathbf{m}^{inv} - \mathbf{m}^c) \parallel^2 . \tag{4}$$

Similarly, $\mathbf{W}_m$ is a diagonal matrix, $\mathbf{m}^{inv}$ denotes the inverted model in each iteration, and $\mathbf{m}^c$ is the so-called confidence map which depends on both the inversion results and the prior information. In practice, these priors are obtained, for example, from well data.

Seismic facies are defined as any observable attribute of rocks such as elastic properties, connectivity and overall appearance over a geological area. Facies can therefore provide rock physics relationships, which can be utilized as constraints in the inversion. This is a key feature (i.e. rock physics constraints per facies) as opposed to assuming only one relationship over the entire area (Kemper and Gunning, 2014; Zabihi Naeini and Exley, 2017).

With a probabilistic type inversion in mind, quantitative FWI can be analyzed from a statistical perspective. Thus, we, specifically, use probability tied to the quantitative inversion results to calculate a confidence map (we plan to extend this to the more advanced Bayesian type inference in the future). To be consistent with the least-square criterion, the Gaussian model is used to describe the uncertainties in the model space (Tarantola, 2005). We evaluate the uncertainties involved in the P-wave, S-wave velocities, and density to generate a confidence map of the possible facies for each parameter. However, individual inverted elastic parameters cannot guaranty to find the correct facies. Hence the multiplication of the uncertainties for each elastic parameter is proposed to find the facies with maximum likelihood. To clarify, the inverted parameters comply with the following distribution,

$$\mathbf{W}_{prior} = exp(-\gamma(\mathbf{m}^{inv} - \mathbf{m}^0)^T(\mathbf{m}^{inv} - \mathbf{m}^0)), \tag{5}$$

where $\mathbf{W}_{prior}$ is an uncertainty matrix for the P-wave, S-wave velocities, and density (columns) with respect to the corresponding facies dependent elastic properties $\mathbf{m}^0$. In fact, $\mathbf{m}^0$ has the same dimension as the model space, but driven by the facies. Also, γ controls the resolution of the calculated confidence maps. The confidence map is then calculated as a weighted average of all the facies given,

$$m_i^c = \mathbf{w}^T\mathbf{m}^0. \tag{6}$$

Here m_i^c is the ith element of the confidence map $\mathbf{m}^c$ in equation 4. $\mathbf{w}^T$ are summation weights calculated from column dot product of $\mathbf{W}_{prior}$ in equation 5. The gradient with respect to the three terms of the objective function is written as,

$$\mathbf{g} = \left(\frac{\partial \mathbf{d}^{pre}}{\partial \mathbf{m}}\right)^T \mathbf{W}_d^T\mathbf{W}_d(\mathbf{d}^{pre} - \mathbf{d}^{obs}) + \alpha \mathrm{div}\left(\frac{\nabla \mathbf{m}}{\sqrt{\varepsilon^2 + \parallel \nabla \mathbf{m} \parallel^2}}\right) + \beta \mathbf{W}_m^T\mathbf{W}_m(\mathbf{m}^{inv} - \mathbf{m}^c). \tag{7}$$

Examples

We consider a layered model with six facies as given in Table 1. A staggered finite-difference method is used to solve the elastic equation with absorbing boundary layer conditions. The model size is 1.75 km by 2.5 km with 50 explosive point sources distributed evenly on the surface shown in Figure 1. The recorded seismic data are multi-component particle velocities. The initial model shown in Figure 2 is a smoothed version of the true model obtained by applying a smoothing operator of length 200 m.

***Table 1** Six facies in the model.*

Face number	1	2	3	4	5	6
P-wave vel. (km/s)	2.5	2.8	2.878	3.158	3.189	3.349
S- / P-wave vel. ratio.	0.54	0.53	0.47	0.6	0.57	0.54
Density / P-wave vel. ratio	0.88	0.93	0.8	0.65	0.69	0.67

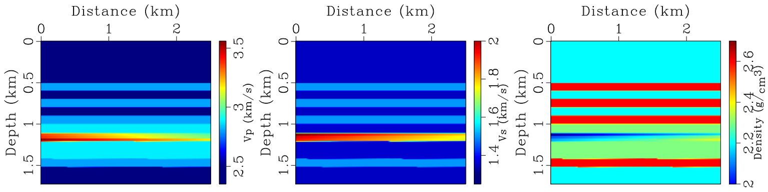

***Figure 1** True models. P-wave velocity (left), S-wave velocity (middle) and density (right).*

Three frequency bands are used in the inversion, which are 2-7 Hz, 2-10 Hz, and 2-13 Hz, sequentially. To be more practical, random noises and low-cut filtering are applied to the observed data and the

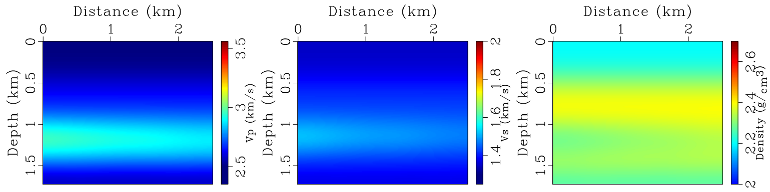

Figure 2 *Initial models. P-wave velocity (left), S-wave velocity (middle) and density (right).*

outcome is shown in Figure 3. We have conducted a standard elastic full waveform inversion as a reference shown in Figure 4 to compare with the results of our proposed method as shown in Figure 5. To assist visual comparison, a vertical profile in the middle of the model is also plotted in Figures 6 and 7. The standard method overestimates the S-wave velocities and there is a relatively strong crosstalk between P-wave velocity and density, as expected. However, the proposed facies based method can recover most of the layers correctly. The normalized total misfit subject to the objective function versus iterations (frequency band 2-13 Hz), shown in Figure 8, reveals that the proposed method does indeed help fitting the data further. The first 13 iterations are standard elastic FWI, after which the facies based regularization term has been activated. Figure 9 shows the observed data, the predicted data and the data residual for the proposed method at 2-13 Hz respectively.

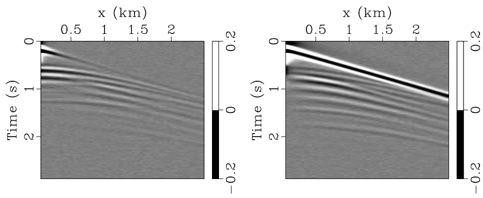

Figure 3 *Observe data at 2-13 Hz. Vertical (left) and horizontal (right) components.*

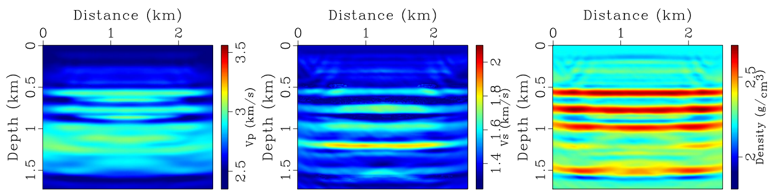

Figure 4 *Standard elastic FWI results. P-wave (left), S-wave (middle) velocities and density (right).*

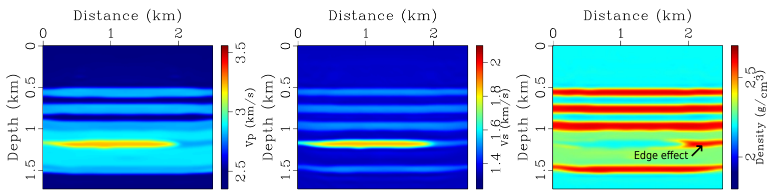

Figure 5 *Proposed elastic FWI results. P-wave (left), S-wave (middle) velocities and density (right).*

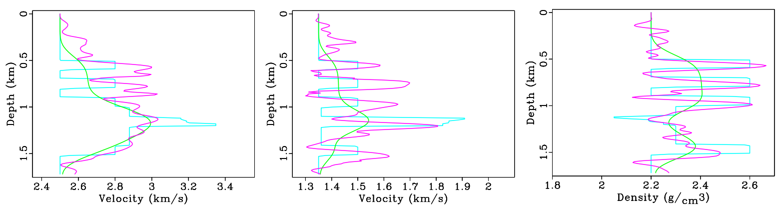

Figure 6 *One vertical profile of the conventional method. P-wave (left), S-wave (middle) velocities and density (right). Cyan: true model; Green: initial model; Pink: standard elastic FWI.*

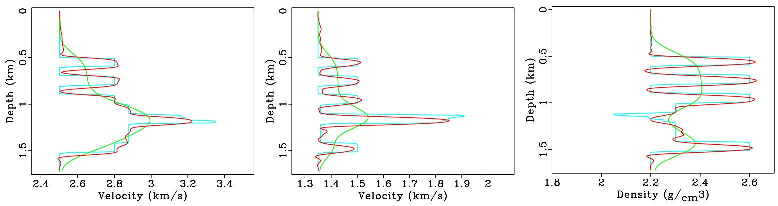

Figure 7 *One vertical profile of our proposed method. P-wave velocity (left), S-wave velocity (middle) and density (right). Cyan: true model; Green: initial model; Red: Proposed method.*

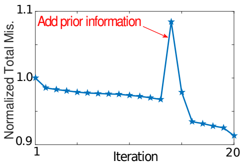

Figure 8 *Normalized total misfit versus iteration at 2-13 Hz. A jump occurs after adding the 3rd term in equation 1.*

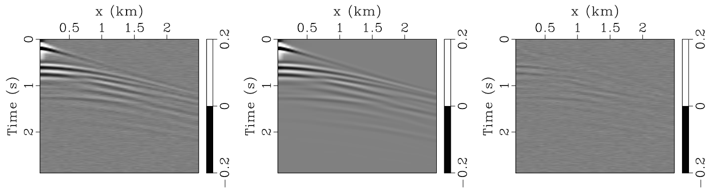

Figure 9 *Observed data with random noise at 2-13 Hz (left); Predicted data of the proposed inversion result (middle); Data residual (right).*

Conclusions

We proposed a novel way to utilize facies dependent prior information to constrain the elastic FWI. A so-called statically driven confidence map is calculated and iteratively updated based on the inversion results and the priors. It is consistent with the framework of the local optimization method, which has an assumption of Gaussian distribution for both model and data uncertainties. The numerical example shows that the proposed method can suppress the cross-talk between different parameters and also can improve the resolution of the estimated elastic properties. However, edge effect and other artifacts can degrade the results as the proposed updates depend on the first pass inversion results. We plan to extend the approach to anisotropic elastic and also investigate the uncertainties involved in the accuracy of priors and its impact on FWI in the future.

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