Introduction

Ask geoscientists, drillers, etc., and they will tell you, unsurprisingly, that the subsurface is modelled and drilled in Depth. However, whether the seismic is (two-way) Time or Depth indexed, seismic inversion products (impedances) are ubiquitously derived in Time, as convolution of an earth model with an appropriate wavelet (an essential step in any inversion) must be done in that domain where the wavelet can usually be assumed stationary. Put a different way, convolution is not easily or naturally represented in Depth, as the effective wavelet shortens or lengthens with varying subsurface velocity, one of the very quantities the seismic inversion attempts to determine. So in the case of Depth indexed seismic, first a Depth to Time conversion must take place. After the inversion is performed in Time, the Time indexed results are usually Time to Depth converted, for use in e.g. geomodelling workflows. Note that the various domain conversions are often concealed from the user. Whilst this approach is awkward (two domain conversions for Depth indexed seismic, a natural product of PSDM or FWI processing), it is so far acceptable for straightforward seismic inversions.

There are however a number of reasons why a new approach in Depth is required. Firstly, facies (or rock-type) based seismic inversion systems have become increasingly de rigeur (Kemper and Gunning, 2014), meaning that not only impedances are derived, but also facies. Whilst subsequent Time to Depth conversion of the impedances (continuous quantities) is feasible, such a domain conversion of discrete facies is not possible without strong aliasing effects. Secondly, for 4D inversions, the natural domain is Depth (we exclude in this discussion cases with significant compaction), as in Time the baseline and monitor surveys again do not align since production (and injection) will have altered the velocity field. Of course to date most 4D analyses are qualitative (e.g. inspection of the quadrature-phase of the seismic difference) but a good quality facies based 4D inversion in Depth should make the analysis more quantitative.

Recasting the convolution operation to Depth is nontrivial. Singh (2012) has attempted this using a stretching technique: for Depth indexed seismic, the Depth axis is stretched and squeezed so that the resultant velocity is constant, and wavelet convolution can then take place safely in this pseudo Depth domain. The convolutional results are then transformed back to the original Depth axis. This is essentially the same as performing a Depth to Time conversion and post inversion a Time to Depth conversion as described earlier. Another approach that gained traction over the last couple of years is the use of Point Spread Functions (PSFs) (Lecomte *et. al.*, 2015). In practice, PSFs are often difficult to obtain, and the 3D character of the operator makes them CPU intensive for inversion schemes. When they are available, we prefer to use them in a lateral deconvolution preprocessing step prior to inversion (Zabihi Naeini, 2018).

In this paper we introduce a new, practical approach to directly obtaining Depth indexed seismic inversion products, both impedances and facies, independent of whether the seismic is Time or Depth indexed. The new method is a modification of the facies-based inversion system of Gunning and Sams (2018). The model is represented in Depth, so no lossy Time to Depth model remapping is required. The Depth model representation has the considerable benefit of allowing regular or irregular gridding, e.g. corner point grids, with stratigraphic alignment in Depth, which marries well with the discrete facies model.

Theory

In voxellised joint facies-elastic inversions, latent facies variables F are inferred which implicitly define a background low frequency model (LFM) of elastic parameters, using rock physics loading models which are facies specific. Using an elastic model, $m = \{v_p, v_s, \rho\}$, the inversion aims to maximise the Bayesian posterior probability, $P(m, F|y) \sim P(y|m)P(m|F)P(F)$, where y is the seismic, and $P(y|m)$ is the likelihood of the synthetic seismic generated by the forward model. Here, $P(m|F)$ embeds facies-specific rock physics models, which requires multiple Depth-trended LFMs, one for each facies expected in the subsurface, together with an assessment of within-facies $\{v_p, v_s, \rho\}$ uncertainty. $P(F)$,

is specified by facies proportion estimates and some interaction terms promoting spatial continuity of labels. The inversion machinery maximises the joint posterior probability of the elastic model and labels using the expectation-maximisation algorithm (Fig 1), involving *soft* estimates of the labels F called *memberships*. Uncertainties can also be estimated using a simulated annealing method to sample multiple equi-probable realisations of impedances and facies from the posterior distribution (Gunning and Sams 2018, Waters and Kemper 2018), but this is not further discussed here.

Figure 1 The Expectation-Maximisation method. In iteration 1 of the Maximization step, the LFM is the membership weighted average of the input LFM's. The misfit between synthetic and seismic is determined as part of the Maximisation step. In the Expectation step, a 3D discrete Markov random Field is used, in which spatial information is embedded; this makes the approach a geostatistical inversion system (as well as a Bayesian one). Note though that variography is not used (not recommended for discrete quantities).

Since the underlying model representation is in Depth, the forward model step in the maximisation step requires a velocity-consistent, on-the-fly mapping of reflectivity to the Time domain. With smart interpolation techniques applied before the convolution operation, this mapping is essentially aliasing free. The following pseudo algorithm describes how Depth based joint impedance and facies inversion is implemented.

1. If the seismic is Depth indexed – perform a Depth to Time conversion (if the seismic is Time indexed, do nothing). Set initial memberships to proportions estimates.
2. M-step: Optimise the expected log-posterior distribution, which is a weighted average of data misfit terms and membership-weighted elastic prior terms. The forward model in the data misfit maps reflectivity self-consistently to Time before convolution. Amplitude misfits are accrued in the Time domain.
3. E-step (entirely in the Depth domain): from the current model, recompute the facies memberships based on the elastic prior distribution misfits, prior proportions, and continuity terms in the prior facies distribution, $P(F)$.
4. Alternate steps 2, 3 till convergence.

Velocity consistency. An issue of importance is what velocity model to use in the various domain conversions. By combining seismic velocities, checkshots and velocity logs, the user typically derives, as part of any inversion project, a kinematic velocity model (from 0 to a few Hz) for general purposes, e.g. to switch the model display between Depth and Time, or as a LFM for simple inversions. For consistency, it is recommended that this velocity is also the one used to map Depth indexed seismic to Time for a Time domain wavelet estimation/well-tie step.

There is however another velocity volume, the one associated with the evolving inversion model under the EM algorithm. This seismic amplitude-driven v_p image contains frequencies from 0 Hz up to the Nyquist frequency based on the TWT sample rate of the seismic. Over the low frequency range this should be consistent with the general purpose kinematic velocity field just described. To enforce this required consistency between the kinematic and amplitude-driven velocities, a kinematic misfit term can also be introduced into the misfit function in the M-step. The resultant optimised v_p field is then a balance between fitting the amplitudes and fitting the kinematic velocities; most typically we see the misfit terms act on different parts of the velocity spectrum, so the terms do not significantly compete. The most likely source of contention here is inconsistency between the shale-facies velocity models and the "general purpose" kinematic velocity field.

Case Study

The dataset utilised for this study is from the Forties field, in the UKCS, one of the largest fields offshore UK. The data comprises 5 angle stacks (9-42 degrees) and approximately 30 exploration and appraisal wells. Even though processed through a pre-stack Depth migration workflow in 2010, the seismic datasets were only available in Time, complete with a suite of TWT seismic interpretations at the key stratigraphic markers.

As per the workflow described earlier in this paper, a velocity model was constructed by a combination of average and interval velocity mapping to each of the key target horizons. This calibrated velocity model is then utilised throughout the inversion process to ensure consistency between the Time and Depth domains.

The facies based inversion was first parameterised to derive optimal results in the Time domain, using the native TWT seismic, sampled at 4ms. Four Time horizons were used in the construction of the stratigraphic model and zonation. Five angle dependent Time domain (4ms sampling) constant phase wavelets were derived from the seismic data for use in the inversion. Within each zone, prior proportions of each of the key 'elastic' rock types, namely 'soft shale', 'hard shale', 'brine sand' and 'oil sand' were defined. An initial inversion was performed, after which some iteration was required to optimise the facies proportions such that adequate results were achieved at wells and along key lines of section. Optionally the Time indexed results can be converted to Depth using the velocity model (see Fig. 2, right hand side)

Once the Time inversion was finalised, the model was re-parameterised in Depth. The seismic, wavelets and prior proportions along with all other parameters were kept fixed, to ensure a robust comparison between the Time and Depth inversions could later be made. The Time horizons were converted to Depth to create a stratigraphic model equivalent to the one used in the Time inversion. The sample increment selected for the Depth domain model was 3m, which is significantly finer than the Time increment of the seismic. It should be noted that although the model is defined in Depth, the convolution is still performed in Time (at the target wavelet sampling of 4ms), as the reflectivity series are Depth to Time converted on the fly using the velocity model.
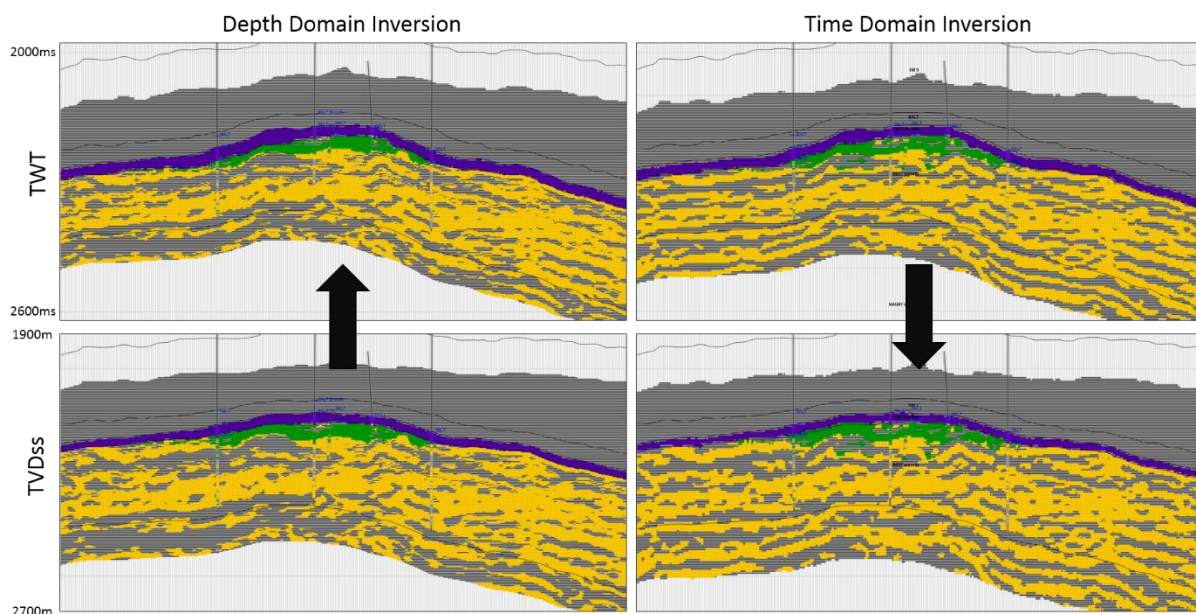

Figure 2 *Left: Depth domain inversion (bottom) and converted to Time (top). Right: Time domain inversion (top) and converted to Depth (bottom). The arrows indicate the 'direction' of domain conversion performed after the inversion.*

Figure 2 shows that a Depth based inversion shows a clear uplift in image clarity and quality, with mis-positioning and jitter of the facies labels significantly reduced or removed, compared to the Time domain inversion. We would like to stress that it is possible, as shown, to generate high resolution Depth models, whilst honouring the sample rate of the seismic in Time.

Conclusions

The Depth domain inversion described in this paper has a number of advantages over conventional Time-domain inversions. It results in an improved velocity model, consistent to both the amplitude and kinematics information (which may be very important e.g. to Time/Depth convert flat structures, or as a starting model for FWI). Furthermore a Depth domain inversion provides the correct framework for quantitative 4D inversion, as with production the seismic Time changes (as mentioned, we assume that compaction is small). Finally inversion results derived in Depth (at a much finer sampling increment than can readily be derived in Time) allow for better reservoir characterisation and improved linkage to geomodelling and flow simulation workflows.

References

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