Image- and horizon-guided interpolation

Ehsan Zabihi Naeini and Dave Hale

ABSTRACT

Seismic data are uniformly sampled images of geologic structures, and they spatially cover a big area as opposed to well log data, which are often scattered sparsely within the seismic cube. A common strategy to extend the coverage of the well log data is to interpolate them away from the boreholes on the seismic image grid. Image-guided interpolation is a form of blended neighbor interpolation in which the structural information that guides the interpolation is calculated from a seismic image and represented by structure tensors. Deriving this structural information from only the seismic image might not lead to desirable interpolated results, especially in cases of complex structure, a low-quality seismic image, and few well logs. In such cases, constraining the structure tensors with additional information is beneficial. We have developed a three-step hybrid method in the log-Euclidean domain to use interpreted horizons as well as the seismic image to construct the final structure tensor field. Also, we determined how to incorporate the curvature attributes from horizons for better performance of the algorithm in the presence of faults. Synthetic and real data examples determined the ability of this new technique to achieve better conformance of interpolated properties with the seismic image and interpreted horizons.

INTRODUCTION

Interpolation of borehole data into a uniformly sampled 3D grid is often used in seismic data processing (Bachrach and Osypov, 2014; Zhou et al., 2014) and reservoir characterization workflows (Pillet et al., 2007; Douma and Zabihi Naeini, 2014). Traditionally, interpolation algorithms have often been guided by horizon surfaces (Hampson et al., 2001; Pedersen-Tatalovic et al., 2008; Huck et al., 2010), although more recently, a new algorithm has been introduced that uses the seismic image as a guide to interpolate borehole data over the entire desired grid (Hale, 2009a). One fact to consider is that when the interpolation is guided by interpreted horizons alone, it will honor the data at locations on these surfaces, but the problem is that the interpolated field will be unable to recover structural details between the horizon surfaces. Also, any mispicks on any of the surfaces would be scattered into the interpolated field. On the other hand, the image-guided interpolation technique has the ability to conform interpolated properties to the seismic structure. This process could fail, however, in cases of complex geology, low-quality seismic, or few wells. The ideal solution seems to lay somewhat in the middle in which the interpolation is to be guided by the seismic- and human-interpreted horizon surfaces.

It is often the case for most interpolation algorithms to calculate some measure of distance from every grid point in which the interpolated values are desired to the known points (borehole data in this paper). The non-Euclidean distance measure using the seismic image is introduced by Hale (2009a), and the interpolation algorithm is called the blended neighbor interpolation. The resulting interpolated field conforms to the features and orientation of the seismic image. This, in other words, is seismic image-guided interpolation (Note that the classic natural neighbor interpolation is based on the Euclidean distance measure and can be obtained by discarding the seismic image in the blended neighbor interpolation algorithm).

In this paper, we show how to use a seismic image and horizon surfaces to guide blended neighbor interpolation. In our proposed algorithm, we maintain the benefits of image-guided interpolation as discussed by Hale (2009a), but we also benefit from other available information, in this case, interpreted horizon surfaces. We start by mathematically explaining the blended neighbor interpolation algorithm. This will be followed by describing the calculation of structure tensors from a seismic image and from horizon surfaces. We then propose an algorithm to create a hybrid tensor field from the seismic image and horizon surfaces. Using the resultant tensor field in the blended neighbor algorithm leads to what we call image- and horizon-guided interpolation. The advantages of this new
technique on various data examples are demonstrated and compared with image-guided interpolation.

**IMAGE-GUIDED INTERPOLATION**

Let us assume that the spatially scattered data (well log values in this paper) to be interpolated are a set

$$F = \{f_1, f_2, \ldots, f_K\}$$

of $K$ known sample values $f_k \in \mathbb{R}$ that correspond to a set

$$\mathcal{X} = \{x_1, x_2, \ldots, x_K\}$$

of $K$ known sample points $x_k \in \mathbb{R}^n$. Together, these two sets comprise a set

$$\mathcal{K} = \{(f_1, x_1), (f_2, x_2), \ldots, (f_K, x_K)\}$$

of $K$ known samples. These samples may be scattered such that the $n$-dimensional sample points in the set $\mathcal{X}$ may have no regular geometric structure. The classic interpolation problem is to use the known samples in $\mathcal{K}$ to construct a function $q(x): \mathbb{R} \rightarrow \mathbb{R}^n$, such that $q(x_k) = f_k$. Hale (2009a) facilitate image-guided interpolation by formulating the blended neighbor interpolation in two steps, as follows.

Step 1 is to solve the Eikonal equation:

$$\nabla t(x) \cdot D(x) \cdot \nabla t(x) = 1, \quad x \notin \mathcal{X};$$

$$t(x) = 0, \quad x \in \mathcal{X};$$

for $t(x)$, which is the minimal time from $x$ to the nearest known sample point $x_k$, and for $p(x)$, which is the value $f_k$ corresponding to the sample point $x_k$ nearest to the point $x$, hence resulting in the nearest neighbor interpolation.

Step 2 is to solve the blending equation:

$$q(x) = \frac{1}{2} \nabla \cdot \nabla t^2(x) D(x) \cdot \nabla q(x) = p(x)$$

for $q(x)$, which is the blended (smoothed) neighbor interpolated field.

In equations 4 and 5, $D(x)$ is a metric tensor field containing spatially varying coefficients of both equations. This results in time $t(x)$ from equation 4 to be a non-Euclidean distance measure; that is to say, $t(x)$ varies with direction. The $D(x)$ together with $t(x)$ in equation 5 control the extent of smoothing of nearest neighbor interpolated values. It is, therefore, the tensor field $D(x)$ that plays the key role in how we desire to guide the interpolation engine, whether it is seismic image guided, horizon surface guided, or a hybrid combination of both. In the next section, we briefly explain the calculation of the metric tensor field for image-guided interpolation.

**Computing the metric tensor field from a seismic image**

Seismic images provide an excellent source of information about the structure and the stratigraphy of the earth. When $D(x)$ is computed from a seismic image, all sample points within a geologic layer (e.g., along coherent events) are enforced to be “near” in the time map $t(x)$ and they are enforced to be “far” when they are in different layers (across coherent events and faults); $D(x)$ also controls the extent and the direction of smoothing in the blending equation.

Structure tensors, as described by Fehmers and Höcker (2003), are the smoothed outer product of image gradient vectors, and they mathematically provide information about the orientation of local features in the image. Hale (2009a) uses the modified inverse of the structure tensors as a measure for the metric tensor field $D(x)$. In 3D, each tensor for each sample is a $3 \times 3$ symmetric positive-definite (SPD) matrix with eigendecomposition:

$$D = \lambda_u u u^T + \lambda_v v v^T + \lambda_w w w^T,$$

where $\lambda_u$, $\lambda_v$, and $\lambda_w$ are the eigenvalues and $u$, $v$, and $w$ are the eigenvectors. Note that by inverting the structure tensors, eigenvectors remain the same (and still point to the local orientation) but only eigenvalues are inverted. Hale (2009a) further modify the eigenvalues so that they are all normalized between zero and one. One can see that the metric tensor field $D(x)$ has different eigenvalues and eigenvectors for each sample location, and hence they lead to non-Euclidean distance measure as mentioned in the previous section.

**HYBRID IMAGE- AND HORIZON-GUIDED INTERPOLATION**

As described in the previous section, blended neighbor interpolation is guided by spatially varying coefficients of the tensor field $D(x)$ derived from a seismic image. In 3D, these types of tensors can be thought of as 3D ellipsoids. Image-guided interpolation may not produce optimal results in the case of a low-quality seismic image, insufficient known samples (e.g., borehole data), or complicated geology. Interpreted horizon surfaces can, therefore, be an extra source of guidance for interpolation, but first, we must compute structure tensors from them. The primary advantage of incorporating horizon surfaces is likely to be in forcing the interpolated field to follow the structure defined by the surface; however, as will be shown later, one can also extract some information about local discontinuity by using curvature attributes derived from the horizon surfaces. This is useful in the presence of faults. In the next sections, we first propose how to compute the structure tensors from only the horizons and then we describe the computation of a hybrid tensor field from the seismic and all the available horizon surfaces simultaneously.

**Computing the metric tensor field from horizon surfaces**

**Step 1: Computing the metric tensor for each horizon surface**

The eigenvector $u$ of the tensor field $D(x)$ in equation 6 indicates, for each sample, the direction in which the image gradients are the highest, in other words, orthogonal to the local 3D plane. In the case of a known horizon surface $z = f(x_1, x_2)$, we can compute the surface normal vector $u_f$ as

$$u_f = a \begin{bmatrix} \frac{1}{2} \frac{\partial f}{\partial x_1} \\ -\frac{\partial f}{\partial x_1} \end{bmatrix}.$$. (7)
where \( a \) is a normalization scalar, and the two partial derivatives are effectively horizon dips in the inline and crossline directions.

To complete the metric tensor field for each sample point of a horizon surface, the other two eigenvectors and the corresponding eigenvalues must also be defined. We address the problem by assuming that we are only interested in the structural shape of the surface, i.e., to constrain the interpolation to follow the horizon surface. In that case, we define the horizon metric tensor as

\[
D_f = \lambda_{fu} u_f u_f^T + \lambda_{fv} v_f v_f^T + \lambda_{fw} w_f w_f^T,
\]

where we set the eigenvalues such that each tensor forms a flat circular shape (e.g., like a coin). This can be achieved by setting \( \lambda_{fu} = 0.0001 \), \( \lambda_{fv} = 1.0 \), and \( \lambda_{fw} = 1.0 \) and defining eigenvectors \( v_f \) and \( w_f \) in any arbitrary direction orthogonal to \( u_f \), and also orthogonal to each other.

**Step 2: Populating the volume with the computed tensors**

The 3D blended neighbor interpolation requires a tensor field calculated at every sample location in the volume. In step 1, we show how to compute the metric tensor field for each sample on the horizon surface. To populate the whole volume, we propose interpolation (and extrapolation) of the metric tensor fields calculated at each of the horizon surfaces to every required output sample location. For example, assume there are two available horizon surfaces and, therefore, the metric tensor for every sample in between the two surfaces must be computed. One might perform a simple 1D interpolation for every element of the tensor (e.g., nine interpolations for a 3\( \times \)3 tensor). For example, in the schematic example shown in Figure 1, structure tensors are calculated at horizons one and two and are set to be fixed above and below them. Between the two horizon surfaces though, every element of the tensor field is a result of interpolation between two horizons. Such an approach leads to an undesirable result as shown in Figure 1a. This is known as the swelling effect in the medical imaging domain (Note that we set the first eigenvalue to be small, which results in very thin ellipsoids. This shape has swelled as a result of interpolation between the two horizons). It turns out that mathematical operations such as averaging, filtering, and interpolation on tensor fields are not quite as straightforward as one would expect. The metric tensors as defined in this study are SPD and need to remain so under any such operations. We therefore adopt the log-Euclidean metrics (Fillard et al., 2005) in which such operations are applied in the log-Euclidean domain. This means that the logarithm of each tensor is first needed before interpolation. This can be computed through eigenvalue decomposition as

\[
D_f = R^T \Lambda_f R \quad \log D_f = R^T \tilde{\Lambda}_f R,
\]

where \( R \) is the rotation matrix (contains eigenvectors) and \( \Lambda_f \) and \( \tilde{\Lambda}_f \) are the diagonal matrices whose diagonal elements are the eigenvalues and the logarithm of the eigenvalues correspondingly. This simply states that the logarithm of a tensor is obtained by recomposing back the tensor using the logarithm of the eigenvalue matrix.

After this transformation into the logarithmic domain, we interpolate the tensor elements followed by matrix exponentiation (i.e., recomposing the metric tensor using the exponentiation of the eigenvalue matrix) to derive the final metric tensor field from all available horizon surfaces. Figure 1b shows how this approach effectively eliminates the swelling effect, and the interpolated tensor field now preserves the shape between the two horizons. Figure 2 compares this process on a real data example in which the improvement in removing the swelling effect is shown by red arrows. A magnification is shown on the right of each figure to assist visual inspection. The horizon-based structure tensor field can be used directly in the blended neighbor interpolation algorithm, and this results in horizon-guided interpolation. We show an example of this in the “Real data examples” section.

**Step 3: Blending the metric tensors derived from seismic and horizon surfaces**

Let us remind ourselves that the objective is to compute a hybrid metric tensor field to improve the performance of blended neighbor interpolation. We perform this by combining the metric tensor field derived from the seismic image (see the “Image-guided interpolation” section) with the metric tensor field derived from the horizon surfaces after step 2. The result is called the hybrid tensor field in this study.

Blending of the two tensor fields can be achieved by weighted averaging. The corresponding weights, normalized to one, can be defined so that they respond to the quality of the seismic image and the accuracy of the interpreted horizons. One option is to choose the weights to be based on seismic structural oriented semblance (Hale, 2009b). The semblance-based attribute is normalized and is close to one when the seismic coherency is high. Alternatively, one can choose Euclidean distance measures from each output sample location to the nearest sample on a horizon surface as a weight for the horizon tensor fields. That means the weights for the horizon tensor field are inversely proportional to the distance from the nearest known horizon sample (as we force the sum of the weights to

Figure 1. Schematic display showing how to populate an image with the tensor field calculated at each horizon surface. Above horizon 1 and below horizon 2, the tensors are set to be fixed. Between the horizons, the tensor field is obtained from interpolation. (a) The swelling effect associated with interpolation of tensors between two horizons, and (b) log-Euclidean interpolation eliminates the swelling effectively.
be one, the complementary weight is chosen for the seismic tensor field). Measuring the distance in this manner is often known as the distance transform in the literature. Maurer et al. (2003) introduce a fast algorithm to compute such a distance measure in arbitrary dimensions. One could also choose a mixture of coherency- and distance-based attributes for combining the two tensor fields. In fact, there is freedom to choose any arbitrary attribute; the above-mentioned attributes are only proposed because we have found them useful for this purpose. Figure 3 shows a comparison of these three different attributes.

After the blending weights are computed, we use them in the blending equation below, using the log-Euclidean domain:

$$D_{\text{final}}(x) = \exp\left(w_{\text{seismic}} \log D(x) + w_{\text{surface}} \log D_f(x)\right).$$

(10)

where $w_{\text{seismic}}$ and $w_{\text{surface}}$ are the corresponding weights for the seismic and horizon surfaces ($w_{\text{surface}} = 1 - w_{\text{seismic}}$), and $\exp$ is the matrix exponential. Figure 4 (zoom display of the same seismic image as in Figure 2 to aid visual comparison) shows a comparison of the metric tensor field calculated from the seismic image ($D(x)$) in equation (10), horizon surfaces ($D_f(x)$), and the final hybrid product $D_{\text{final}}(x)$. As one would expect, the observation here is that the interpreted horizons help to derive an optimal tensor field. The impact on the interpolation will be discussed in the next section.

**SYNTHETIC DATA EXAMPLE**

Figure 5a shows a synthetic seismic image with flat unconformity separating a flat structure on top from a more complex faulted structure underneath. Four wells with real well log values inserted in the 3D synthetic seismic cube to simulate the interpolation more realistically (therefore, the well log values do not correspond with the underlying reflectivity sequence used to generate the synthetic). The seismic image shown in Figure 5a intersects with two wells. Figure 5b is the result of image-guided interpolation of acoustic impedance (AI) (the units are g/m/cm³ throughout this paper) values, and Figure 5c is from hybrid image- and horizon-guided interpolation. The latter uses the horizon as well as the seismic image. The seismic image encounters rapid changes just below and above the unconformity, and this in return leads to instability in the structure tensors and hence the interpolated field. Although the unconformity is flat, it helps to stabilize the interpolation results significantly around the horizon as shown in Figure 5c.

The impact of using the horizon as well as the seismic is more evident when viewed on 3D slices. Figure 6a and 6d shows time slices just above and just below the unconformity accordingly. As expected, the time slice above the unconformity horizon is constant, which corresponds to a flat structure. Figure 6b and 6c compares the same time slice just above the horizon from the two interpolation techniques. It can be observed that using only the seismic image results in an interpolated field above the horizon with the footprint of the structure below the horizon, which is clearly not correct. Using the horizon, though, overcomes this issue, and the interpolated field correctly represents values weighted with an inverse of the distance to the nearest well. Time slices below the horizon,
Figure 6e and 6f, show that both techniques correctly image the structure of the seismic, although the actual interpolated values are accurate only in the case of hybrid image- and horizon-guided interpolation. At deeper parts below the horizon, both techniques are identical, as desired.

REAL DATA EXAMPLES

Example 1

In Figure 7, there is an example using the same seismic image shown in Figures 2 and 3. In this case, we have two interpreted horizons and one well. There are three distinct geologic packages: a thick homogenous soft shale unit below the first horizon, which is overlying reservoir sands and intrareservoir shales, and then beneath that is a chalk unit (below the second horizon). Therefore, it is of great importance to achieve an interpolated property that honors the geology, stratigraphy, and interpretation. Figure 7 shows the comparison between the image-guided interpolation and the hybrid interpolation in which we can observe that the optimal result is obtained especially in the areas around the second horizon where the chalk unit is separated from the reservoir and intrareservoir shale units above. There are also other interesting features in the hybrid interpolated field; for example, the pinch-out body intersecting the well between the two horizons marked by an ellipse in Figure 7b. This pinch-out seems to be imaged better using the hybrid approach as the well indicates that the intervals above and below the body are the same layer (e.g., same AI values). Of course, fully detailed analysis and interpretation of these geologic features are beyond the scope of this paper, but one could see that the unique benefit of the proposed technique is the ability to navigate between different possibilities.

Example 2

This example is from the same area as in example 1 with the same geology and structure, although in this case, a 3D subset of the seismic cube around the main reservoir is extracted in which there are

Figure 5. (a) Synthetic seismic image, (b) image-guided interpolation, and (c) image- and horizon-guided interpolation of AI log values. Interpolated values around the horizon are not stable and leak below and above the unconformity when only the seismic image is used for interpolation. Using the seismic and the horizon helps to resolve this issue (see also Figure 6). Note that the AI well log values are real well data and are used here to simulate a more realistic interpolation scenario (that is why the interpolated properties at either side of the fault coming from the nearest well do not match when shifted with the fault throw).

Figure 4. Computed metric tensor field from (a) only the horizons, (b) only the seismic image, and (c) hybrid combination of the seismic and the horizons using the blending attribute in Figure 3d. Note, for example, that the tensors in the areas shown by arrows are optimal from the hybrid solution because the corresponding tensors from only the seismic image are not well aligned with the local features potentially due to the impact of noise.
more wells available. The 3D map views help to better justify the benefits of image- and horizon-guided interpolation and that is the objective of analyzing the performance of the two techniques here. Figure 8a shows a line from the seismic cube where the locations of the time slice and horizon extraction are also indicated. Figure 8b shows the time slice and the seismic extracted just below the horizon at locations shown with a white arrow in Figure 8a. It can also be seen that there are four interpreted horizons available (shown by the pink, orange, purple, and blue lines in Figure 8a), and they all have been used in the image- and horizon-guided interpolation.

Figure 6. (a) Time slice above the unconformity horizon shown in Figure 5, which shows only a constant value because of horizontal layering above the unconformity, (b) image-guided interpolation, and (c) image- and horizon-guided interpolation of AI log values. (d) Time slice below the horizon, (e) image-guided interpolation, and (f) image- and horizon-guided interpolation of AI log values. The dashed line shows the position of the line displayed in Figure 5.

Figure 7. (a) Image-guided versus (b) hybrid image- and horizon-guided interpolation of AI well logs. It can be observed that the interpolated AI values along the second horizon are consistent with the well log using the hybrid approach, whereas the image-guided interpolation cannot track the geology in some areas; e.g., see the arrow where the red color, expected to be above the horizon, leaks into the formation below the horizon. The black box shows a magnification.
algorithm. The results of image-guided and the hybrid image- and horizon-guided interpolation of AI values from 12 wells (the circle symbols on the map view show the spatial location of the wells) are shown in Figure 8c and 8d. In the time-slice view, it can be observed that image- and horizon-guided interpolation has a better performance in providing a more coherent structure as shown by the white arrows. As has been observed in examples so far, interpreted horizons help to get an interpolated field that better conform to the horizons and also avoid leaking the properties to the structure below and above the horizon. The horizon extraction maps in this example restate this in the area shown by a square box. A line through this box is shown in Figure 9. It can be observed that image- and horizon-guided interpolation confines the high impedance values above the horizon and avoid leakage to the formation below. Overall, according to the observations from Figures 8 and 9, this technique benefits from the seismic and horizons and the interpolated field is better conformed to the geologic structure.

Example 3

This example features a more complex geology consisting of an unconformity and various fault blocks. Here, we not only analyze the performance of the proposed methodology on a different data set with different geology, but we also demonstrate how we can potentially extend the benefits of using interpreted horizons by incorporating their curvature attributes (Note that so far, we only used the structural orientation of the horizons). This can be helpful at fault locations with a big throw in which the interpreted horizons often exhibit a high curvature. To close the loop in our study, we also show the horizon-guided interpolation on its own for reference as well as the image-guided interpolation and work toward progressively improving the results with the hybrid approach.

In this example, there are two wells available and also two interpreted horizons, as shown in Figure 10a. The top horizon is an unconformity, and within the interval between the top and bottom horizons, the stratigraphic layering is parallel to the basal horizon. Therefore, it is indeed possible to constrain the metric tensor field from the horizons to honor this feature. Both horizons had been obtained using automatic tracking applications guided by an interpreter. The only problem is at the fault locations, where the horizons are continuous with high curvature. Ideally, one might argue that all of the horizons need to have a gap when intersecting with the faults, but this is not the case here and also in many practical applications (Note that if the horizons had a gap at fault locations, the algorithm works just as fine and there is no need to incorporate the curvature). Figure 10a shows the seismic, interpreted horizons, and AI well logs. Figure 10b is a repeat of Figure 10a with the main faults interpreted as dashed lines for reference. In the following figures, the

![Figure 8](image-url)  
Figure 8. (a) Seismic image, (b) seismic time slice and horizon extraction maps, (c) image-guided interpolation, and (d) hybrid image- and horizon-guided interpolation. Time-slice view shows a more coherent structure from the hybrid approach (for example, see the structure shown by the white arrows). Similarly, horizon extraction maps also show some differences notably in the square box; see Figure 9 for further analysis.
Figure 9. A line through the box shown by the white dashed line in Figure 8c and 8d. (a) Seismic image and the four horizons used in this example as shown by pink, orange, purple, and blue colors, (b) image-guided interpolation, and (c) hybrid image- and horizon-guided interpolation. The anomalous high impedance values in horizon extraction view in Figure 8c are a result of leakage below the blue horizon in the image-guided interpolation shown by the white arrow here. Again, image- and horizon-guided interpolations help to achieve a better conformation of the interpolation properties to the seismic image and horizon interpretation. Note that all four horizons are used for image- and horizon-guided interpolation.

Figure 10. (a) Input seismic image overlaid with two interpreted horizons and two wells with AI logs. Panel (b) is the same as (a) but with major faults indicated with white dashed lines; in the following figures, the location of these faults is shown by white arrows. (c) Image-guided interpolation, (d) horizon-guided interpolation, (e) hybrid image- and horizon-guided interpolation, and (f) hybrid image- and horizon-guided interpolation constrained by the curvature. The low quality and the complex nature of the seismic image cause the leakage shown by the white ellipses in Figure 10c. Horizon-guided interpolation does not suffer from this problem but also does not exhibit any geologic features that are not in the horizons. When horizons are combined with seismic to guide the interpolation, it provides an optimum solution shown in Figure 10e. Hybrid image- and horizon-guided interpolation can be further improved at locations such as in the square box by incorporating the curvature of horizon surfaces to constrain the smoothing of the interpolated values across the fault.
locations of these faults are shown by white arrows for better visual inspection. Figure 10c displays the image-guided interpolation, whereas in Figure 10d, we display the horizon-guided interpolation. As one expects, the image-guided interpolation exhibits the features in the seismic section better and that is desirable at the locations of faults (white arrows). However, in the areas shown by ellipses where the geology is complex and the seismic has a low signal-to-noise ratio, solely relying on the seismic image is not sufficient because the interpolations produce leaks below the basal unconformity horizon. In such cases, as has also been observed in the previous examples, human interpretation becomes very valuable. Figure 10d already gives a hint that using the interpreted horizons avoids the leak below the second horizon. Figure 10e shows the hybrid image- and horizon-guided interpolation in which not only are the faults recovered (compared to Figure 10d), but also the interpolated AI values above the second horizon do not leak below. Although the hybrid approach provides satisfactory results, there is more that can be done. In the next section, we show how we can overcome this issue by incorporating curvature from horizon surfaces.

**Incorporating curvature**

For a surface $z = f(x_1, x_2)$, the Hessian is defined as below

$$H(f) = \begin{pmatrix} f_{x_1x_1} & f_{x_1x_2} \\ f_{x_2x_1} & f_{x_2x_2} \end{pmatrix},$$

(11)

where the subscripts denote the derivative order; e.g., $f_{x_1x_1}$ is the second derivative relative to the $x_1$ (e.g., inline) and so on. From differential geometry (Goldman, 2005), we can show that the mean and Gaussian curvature are defined as

$$K_M = \frac{\nabla f \times H(f) \times \nabla f^T - (|\nabla f|^2 + 1) \times \text{Trace}(H(f))}{2(|\nabla f|^2 + 1)^2},$$

(12)

and

$$K_G = \frac{\text{Det}(H(f))}{(|\nabla f|^2 + 1)^2},$$

(13)

where $K_M$ and $K_G$ are the mean and Gaussian curvature and “Det” denotes the determinant of the matrix.

Principal curvatures measure the maximum $k_1$ and minimum $k_2$ bending of a surface and can be computed from the mean and Gaussian as described below

$$k_1, k_2 = K_M \pm \sqrt{K_M^2 - K_G}.$$  

(14)

In this study, we find the mean and maximum curvature to be most beneficial. The choice of the attribute, however, can vary on different data sets, and the interpreter can choose the right indicator for the faults accordingly. Figure 11a and 11b shows a horizon surface (the second horizon in Figure 10a) and maximum curvature in which one can see the correlation between the curvature map and the possible discontinuities. We then derive a mask from Figure 11b by thresholding the small curvature values to zero and high curvature values to one as shown in Figure 11c. This mask can then be used as a shrinking factor to the metric tensor field so that at the discontinuities, the ellipsoids are forced to be smaller and hence we apply less smoothing. The impact of this on interpolation is shown in Figure 10f where it can be observed that the interpolated values are sufficiently blocked at the fault intersection and do not show the smearing issue shown in the square box in Figure 10e.

**CONCLUSION**

We have shown how to incorporate extra information to update the structure tensors using the log-Euclidean domain. This was used in the context of blended neighbor interpolation to compute a hybrid metric tensor field to guide the interpolation. When compared with image-guided interpolation, we observe that the hybrid use of image and horizon surfaces would generally help to achieve better results. One should note that the goal of this paper is not to suggest that

Figure 11. A map view of the (a) horizon surface, (b) maximum curvature, and (c) maximum curvature after binary thresholding of high curvature values. The fault in the square box in Figure 10e is shown here with a red arrow for reference.
extra horizons must be picked for blended neighbor interpolation, but instead it is to benefit from them if they are available. If the seismic has low quality, then having interpreted horizons would help to stabilize the interpolation. In fact, one could generate different scenarios with varying degrees of the impact of horizons and find the optimal result interactively. We also showed that curvature attributes of interpreted horizon surfaces can be further used to constrain the structure tensors so that the interpolated values are not smeared across faults.

ACKNOWLEDGMENTS

The authors thank Ikon Science for permission to publish this work and Apache North Sea for permission to show the real data example.

REFERENCES


Hale, D., 2009b, Structure-oriented smoothing and semblance: CWP report 635.


